4.8 Using Matrices to Solve Systems of Equations

Matrix Equation

Coefficient Variable Matrix Constant

$$\begin{bmatrix}1 & 3\\ 1 & 2\end{bmatrix}\cdot\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}3\\ 7\end{bmatrix}$$

2x2 2x1 2x1

$$\begin{bmatrix}x+3y\\ x+2y\end{bmatrix}=\begin{bmatrix}3\\ 7\end{bmatrix}$$

$$\begin{cases}x+3y=3\\ x+2y=7\end{cases}$$

Matrix Equation

$$\begin{bmatrix}1 & 3\\ 1 & 2\end{bmatrix}\cdot\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}3\\ 7\end{bmatrix}$$

Coefficient Matrix | Variable Matrix | Constant Matrix

Put the following system into a matrix equation:

5x - 3y = 9
2x + 3y = 7

$$\begin{bmatrix}5 & -3\\ 2 & 3\end{bmatrix}\cdot\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}9\\ 7\end{bmatrix}$$

Use Matrix equation to solve

A-coefficient matrix
X- variable matrix
B- constant matrix

a x = b

$\frac{1}{a}\cdot ax=\frac{1}{a}b$

$1x=\frac{1}{a}b$

$x=\frac{1}{a}b$

A X = B

$A^{-1}AX=A^{-1}B$

$IX=A^{-1}B$

$X=A^{-1}B$

ex:
5x + 3y = 13
4x + 7y = -8

$\begin{bmatrix} 5 & 3 \\ 4 & 7 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 13 \\ -8 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ 4 & 7 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 13 \\ -8 \end{bmatrix}$

(5, -4) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -4 \end{bmatrix}$

ex:
5x - 2y = 3
8x + 4y = 3

$\begin{bmatrix} 5 & -2 \\ 8 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$

$\left(\frac{1}{2}, -\frac{1}{4}\right)$ $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ -\frac{1}{4} \end{bmatrix}$

ex:
10x + 5y = 15 → 2x + y = 3
6x + 3y = -6 2x + y = -2

$\begin{bmatrix} 10 & 5 \\ 6 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 15 \\ -6 \end{bmatrix}$

Either ∅ or ∞ # of sol'ns

∅

Do:
1. 2x + 3y = 10
x + 6y = 32

2. 3x + 4y = -1
6x - 2y = 3

ex:
3x - 2y + z = 0
2x +3y - z = 17
5x -y + 4z = -7

$$\begin{bmatrix} 3 & -2 & 1 \\ 2 & 3 & -1 \\ 5 & -1 & 4 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 17 \\ -7 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ -5 \end{bmatrix}$$

HW
p205-206
4-6, 24, 26-28